education institutions. It is also important to explore the potential of digital platforms for scaling up STEM projects in de-occupied territories and regions with limited access to educational infrastructure.

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APPLICATION OF COMPUTER TECHNOLOGIES AND OPTIMIZATION APPARATUS TO CLASSICAL PROBLEMS OF LINEAR ALGEBRA

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Any process of decision making involves selecting from various alternatives. This choice is governed by our desire to make the most effective, the most optimal decision. Thus, at the heart of any decision-making process, be it in engineering or in economics, always lies optimization. Mathematical optimization is one branch of applied mathematics, focused on finding the best solution from a set of feasible alternatives, often subject to constraints. It is foundational in engineering, economics, data science, and operations research. Optimization problems of various types arise in all quantitative disciplines, ranging from computer science and engineering to operations research and economics. Optimization modeling is rather powerful tool, used, including, in classical mathematics, for instance, in linear algebra. Applied linear algebra allows us to take some radically different look at many classical problems of mathematics [3], that is demonstrated in this paper.

It presents basic results of analysis of classical Gaussian elimination method [1; 4] and gradient methods, as well as their variations [2; 5], for solving arbitrary systems of linear algebraic equations. These results have been obtained after testing our own program, written in «Visual Basic for Applications». Namely, we have combined well-known methods from classical algebra and six optimization methods.

Today, with active use of computers in different areas of our life, we have to admit that computer methods give us innovative, truly non-standard, way to look at different problems from classical mathematics. In particular, even the simplest problems of linear algebra from now on are not just routine tasks for students and can be considered from a different angle via modern technologies. Linear algebra is, probably, the most fundamental tool for machine learning, providing indeed powerful and versatile framework for representing, analyzing, and manipulating data. Its broad applicability to truly wide spectrum of machine learning tasks makes it indeed indispensable skill for professionals in the corresponding field.

We have written our own program in «Visual Basic for Applications», that presents some results of comparison between traditional Gauss method and optimization gradient methods, as well as their variations, for solving arbitrary systems of linear algebraic equations. Namely, we have combined well-known methods of classical linear algebra and the next six optimization methods: gradient descent method with adapted step selection, gradient descent method with adapted step correction, modified gradient descent method, gradient method of the steepest descent, and two gradient methods of conjugate gradients: with the help of formulas of Fletcher-Reeves and Polak-Reiber.

Gradient descent methods are big class of optimization algorithms, commonly used in machine learning and other areas of applied mathematics. These methods aim to find local minimum of the corresponding function by iteratively adjusting some parameters in the direction of the steepest descent. The idea behind gradient descent is based on the fact that local minimum of the function under consideration occurs where its gradient is zero. By repeatedly updating some parameters, gradient descent algorithms gradually converge towards one optimal solution.

An essence of this suggested optimization gradient method in relation to linear systems is that solving an arbitrary linear system

{a11x1+a12x2+a13x3+…+a1nxn=b1, a21x1+a22x2+a23x3++a2nxn=b2, a31x1+a32x2+a33x3++a3nxn=b3, …, am1x1+am2x2+am3x3+…+amnxn=bm,

that can be written in the next matrix form

Ax=b,

where

A=a11 a12 a13 … a1n a21 a22 a23 … a2n a31 a32 a33 … a3n ⋮ ⋮ ⋮ ⋮ am1 am2 am3 … amn =Am×n,

b=b1 b2 b3 … bm =Bm×1, x=x1 x2 x3 … xn =Xn×1,

is equivalent to finding minimizer, i.e. such vector x*Rn, for which

fx =fx*,

for the next quadratic functional, called as residual function,

fx=Ax-b,Ax-b=‖Ax-b‖2, A∈Rm×n , bRm , m≤n.

The method is iterative: starting from an arbitrary point x0 (called as initial approximation) in the corresponding Euclidean space, we will subsequently visit (by repeating always the same computation) points x1, x2, …, until we eventually reach a point that is the solution of the system under consideration.

The program consists of two parts: the first is based on classical (the Gaussian elimination) method and the second is built on using various gradient methods. Screenshots of the program's results, specifically, visual demonstrations of implementation of six gradient methods, for the next system of algebraic equations

{x1+3x2+2x3=10, 4x1+3x2+2x3=20,

is presented below. Namely, necessary result of «classical» Gaussian method: the obtained upper triangular form of the system (row-echelon form of the matrix, in which there are non-zero elements above its main diagonal, running from the upper left corner to the lower right corner, and zeros in every position below its main diagonal) (fig. 1), – and results of six variational methods: the obtained solution, accuracy of all necessary calculations, and running time of the corresponding method (fig. 2 – fig. 7).

x1+3*x2+2*x3=10
-9*x2-6*x3=-20

Fig. 1. The obtained upper triangular form of the system under consideration

x1=3,332449
x2=1,538976
x3=1,025984

7,7578398502554E-03 0,015625 сек.

Fig. 2. The obtained solution, accuracy of all necessary calculations, and running time of gradient descent method with adapted step selection

x1=3,331985
x2=1,539338
x3=1,026225

9,98847237550768E-03 0,46875 сек.

Fig. 3. The obtained solution, accuracy of all necessary calculations, and running time of gradient descent method with adapted step correction

x1=3,333261
x2=1,538385
x3=1,02559

9,40288918083727E-03 0,078125 сек.

Fig. 4. The obtained solution, accuracy of all necessary calculations, and running time of modified gradient descent method

x1=3,331985
x2=1,539338
x3=1,026225

9,9907712533262E-03 0,3828125 сек.

Fig. 5. The obtained solution, accuracy of all necessary calculations, and running time of gradient method of the steepest descent

x1=3,331983
x2=1,539339
x3=1,026226

9,99803827385007E-06 8,023438 сек.

Fig. 6. The obtained solution, accuracy of all necessary calculations, and running time of gradient method of conjugate gradients (using formulas of Fletcher-Reeves)

x1=3,331983
x2=1,539339
x3=1,026226

9,99803827385007E-06 24,63281 сек.

Fig. 7. The obtained solution, accuracy of all necessary calculations, and running time of gradient method of conjugate gradients (using formulas of Polak-Reiber)

After testing various systems of linear algebraic equations with the help of our program, we have mentioned the next important facts. The Gaussian elimination method provides solutions extremely rapidly, within fractions of a second, while our proposed optimization method achieves the highest level of accuracy in a short time not for every system. It gives this result for square, symmetric, positive-definite (or positive-indefinite) matrix of the system under consideration. For some systems the program just loops during execution of this optimization algorithm. This fact experimentally proves that effectiveness of this method depends on value of its step.

Roughly speaking, optimization in mathematical sense is difficult process of finding the best decision, with regard to some criterion, from some set of available alternatives. Active computerization of our life generates new and new optimization problems.

Optimization modeling is rather powerful tool, used in various fields, including operations research, engineering, economics, finance, logistics. Along with this, mathematical apparatus of optimization finds its reflection in various classical problems of linear algebra.

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ФОРМУВАЛЬНЕ ОЦІНЮВАННЯ В STEM-ОСВІТІ: ВИКОРИСТАННЯ ІНТЕРАКТИВНИХ ЦИФРОВИХ ІНСТРУМЕНТІВ НА УРОКАХ МАТЕМАТИКИ Й ІНФОРМАТИКИ

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Активне впровадження STEM-освіти в українських школах вимагає концептуальних змін не лише в методах викладання, але й у підходах до оцінювання навчальних досягнень здобувачів освіти. Традиційне (підсумкове) оцінювання, яке зосереджене переважно на фіксації кінцевого результату, часто виявляється малоефективним і навіть демотивуючим у контексті проєктного навчання.

Для повноцінної реалізації STEM-підходу, де ключову роль відіграють процес пошуку рішення, креативність, алгоритмічне мислення та здатність до командної роботи, необхідний перехід до формувального оцінювання. Як зазначають дослідники, саме формувальне оцінювання — тобто оцінювання «для навчання» — здатне забезпечити безперервний зворотний зв’язок, знизити рівень стресу та підтримати учня на кожному етапі його освітньої траєкторії [2, с. 14].

Особливої актуальності ця проблема набуває на уроках математики й інформатики, які є фундаментальними складовими STEM-напрямку. Сучасні виклики цифровізації, перехід до змішаних форматів навчання та потреби сучасного покоління учнів вимагають від учителя володіння інструментарієм, здатним зробити процес оцінювання прозорим і мотивуючим. Використання інтерактивних цифрових сервісів стає не просто технічним доповненням, а необхідною умовою формування ключових компетентностей здобувачів освіти [4, с. 53].

Формувальне оцінювання розглядають у сучасній педагогіці як безперервний інтерактивний процес між учителем та учнем. Його головна мета — відстеження особистісного поступу здобувача освіти, виявлення прогалин у розумінні матеріалу та вчасне коригування освітнього процесу.

У контексті STEM-освіти такий підхід дозволяє оцінювати не лише формальне відтворення математичних формул чи синтаксису мови програмування. Значно важливішою стає здатність учня застосовувати ці теоретичні знання на практиці,