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**INTERNATIONAL  
ONLINE WORKSHOP ON  
APPROXIMATION THEORY**

**BOOK OF ABSTRACTS**

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$$\ell_{n+s,1}u := D_1^{2s-2}u|_{x_1=0} - D_1^{2s-2}u|_{x_1=1} = 0, \quad s = 1, 2, \dots, n, \quad (3)$$

$$\ell_{s,2}u := D_2^{2s-2}u|_{x_2=0} + D_2^{2s-2}u|_{x_2=1} = f_s(x_1), \quad s = 1, 2, \dots, n, \quad (4)$$

$$\ell_{n+s,2}u := D_2^{2s-1}u|_{x_2=0} + D_2^{2s-1}u|_{x_2=1} + \ell_s u = f_{n+s}(x_1), \quad s = 1, 2, \dots, n, \quad (5)$$

$$\ell_s u := \sum_{q=0}^{k_s} \sum_{r=1}^m b_{q,r,s} D_2^q u(x)|_{x_2=x_{2,r}}, \quad s = 1, 2, \dots, n, \quad (6)$$

$$0 = x_{2,1} < x_{2,2} < \dots < x_{2,k} = 1, \quad a_p, b_{q,r,s} \in \mathbb{R},$$

$$q = 0, 1, \dots, k_s, \quad k_s < 2n, \quad r = 0, 1, \dots, m, \quad s = 1, 2, \dots, n, \quad p = 0, 1, \dots, n.$$

We construct the operator which maps the solutions of the self-adjoint boundary-value problem with mixed boundary conditions ( $b_{q,r,s} = 0$ ) to the solutions of the investigated multipoint problem.

In the case of an elliptic equation the conditions of existence and uniqueness of the solution for the problem (1)-(6) are established.

- [1] Ya.O. Baranetskij, P.I. Kalenyuk, M.I. Kopach, A.V. Solomko, *The nonlocal boundary value problem with perturbations of mixed boundary conditions for an elliptic equation with constant coefficients. 1*, Carpathian Math. Publ., **11**, (2019), 228–239.
- [2] Ya.O. Baranetskij, P.I. Kalenyuk, M.I. Kopach, A.V. Solomko, *The nonlocal boundary value problem with perturbations of mixed boundary conditions for an elliptic equation with constant coefficients. 2*, Carpathian Math. Publ., **12**, (2020), 173–188.



## Multidimensional analogue of Thron’s theorem about twin parabolic convergence regions for continued fractions

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Branched continued fractions (BCF) is a multidimensional generalisation of continued fractions. Lately, it is intensively developed the theory of BCF of the special form with complex elements

$$b_0 + \prod_{k=1}^{\infty} \sum_{i_k=1}^{i_{k-1}} \frac{a_{i(k)}}{b_{i(k)}},$$

where  $i_0 = N$  is a fixed natural number.

**Theorem.** *Let the elements of the BCF*

$$\left( b_0 + \prod_{k=1}^{\infty} \sum_{i_k=1}^{i_{k-1}} \frac{a_{i(k)}}{1} \right)^{-1} \quad (1)$$

lie in parabolic domains, that is  $a_{i(k)} \in \mathcal{P}_{i(k)}$ ,  $i(k) \in \mathcal{I}$ , where

$$\mathcal{P}_{i(k)} = \left\{ z \in \mathbb{C} : |z| - \operatorname{Re}(ze^{-2i\gamma}) < \frac{2D_k^2}{i_{k-1}} (1 - \varepsilon) \cos^2 \gamma \right\},$$

$$\mathcal{I} = \{i(k) : i(k) = (i_1, i_2, \dots, i_k), 1 \leq i_p \leq i_{p-1}, 1 \leq p \leq k, i_0 = N\},$$

where  $D_{2s} = (1-d)^2$ ,  $D_{2s+1} = d^2$ ,  $0 < d < 1$ ,  $s = 1, 2, \dots$ ; the  $\varepsilon$  is an arbitrary small real number ( $0 < \varepsilon < 1$ ).  
Then

- 1) there exist finite limits of even and odd approximants of the BCF (1);
- 2) the BCF (1) converges if the following series diverge

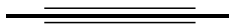
$$\sum_{p=1}^{\infty} |a_{m[p+1]}|^{-1/2}, \quad m = \overline{1, N},$$

$$\sum_{p=1}^{\infty} |a_{i(n), m[p+1]}|^{-1/2}, \quad i(n) \in \mathcal{I}^{(m+1)}, \quad m = \overline{1, N-1};$$

$$\mathcal{I}^{(m+1)} = \{i(k) = (i_1, i_2, \dots, i_k) : m+1 \leq i_k \leq i_{k-1} \leq \dots \leq i_0; k \geq 1; i_0 = N\},$$

$$m[p] = \underbrace{m, m, \dots, m}_p; \quad p = 1, 2, \dots$$

- [1] W.J. Thron, *Two families of twin convergence regions for continued fractions*, Duke Math. J., **10**, (1943), 677–685.



## Generalized Hermite polynomials in the description of fractional calculus and other applications

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Using the concepts and formalism of different families of Hermite polynomials, we here discuss how to represent the action of the operators involving fractional derivatives and we introduce a family of polynomials strictly related to the Hermite polynomials in order to compute the effect of fractional operators on a given function. Finally, we present some generalizations of polynomials belonging to the Bernoulli class. In particular, by using the generating function method and the concept and formalism of the two-variable Hermite polynomials, we introduce the generalized Bernoulli polynomials.



## Two-dimensional generalized moment representations and Pad'e approximations for pseudo-twovariate functions

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Generalized moment representations are used to study the so-called pseudo-twovariate functions

$$f(z, w) = \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \tilde{s}_{k+m} z^k w^m = \frac{z\tilde{f}(z) - w\tilde{f}(w)}{z - w},$$

where

$$\tilde{f}(z) = \sum_{k=0}^{\infty} \tilde{s}_k z^k.$$

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